



Optical resonances in meta-structures for nonlinear  
photonics and color generation

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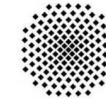
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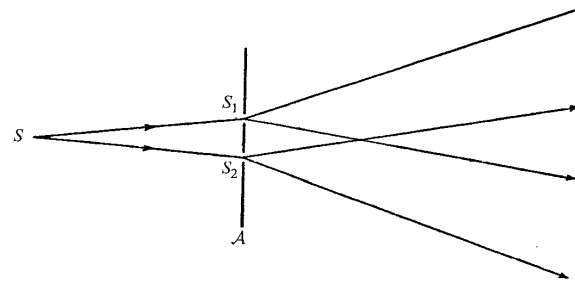
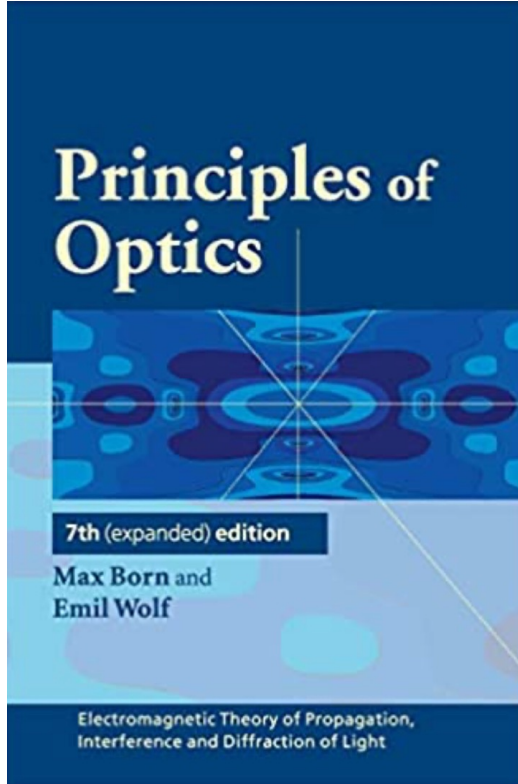


Fig. 7.2 Young's experiment.

VII Theory of interference and interferometers

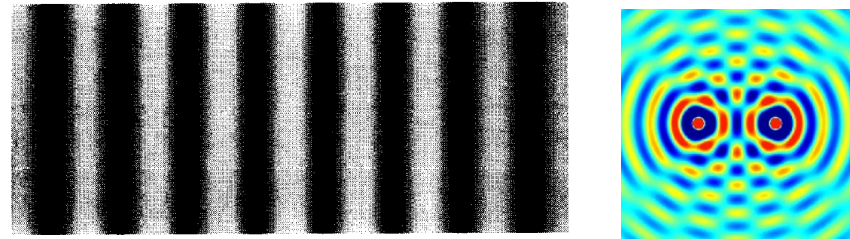
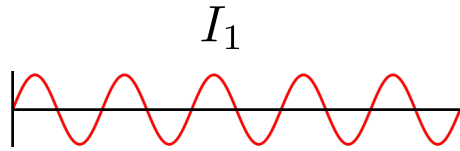
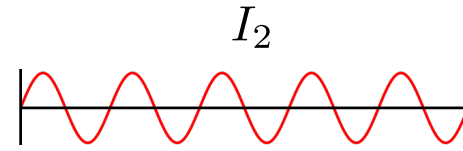


Fig. 7.4 Young's interference fringes.

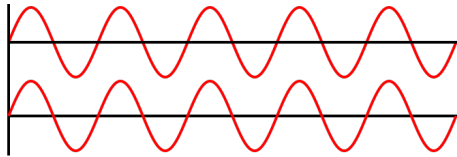
Coherent light beams



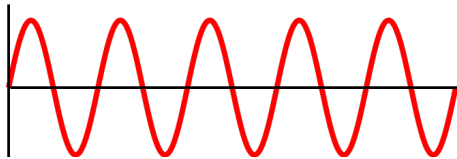
+



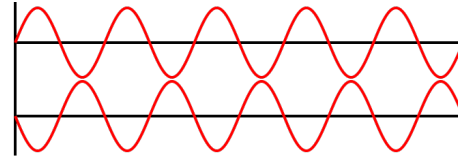
**Constructive interference**



$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}$$



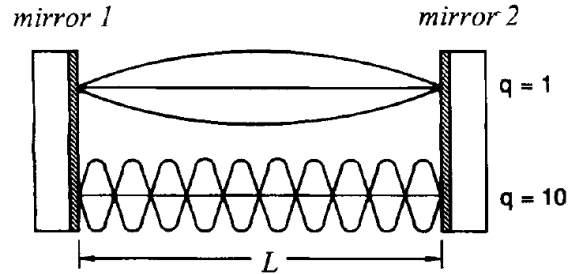
**Destructive interference**



$$I = I_1 + I_2 - 2\sqrt{I_1 I_2}$$



190 Chapter 4 The Fabry Perot Resonator



Hodgson, N. and Weber, H., 2005. *Laser Resonators and Beam Propagation: Fundamentals, Advanced Concepts, Applications*

### Wavelengths of optical resonances

$$\frac{\lambda_q}{2} = \frac{L}{q}$$

$$q = 1, 2, 3..$$

Standing wave = **interference** of contra-propagating waves

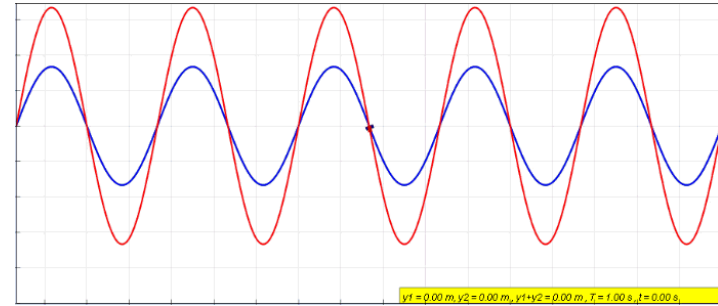


Image: wikipedia.org

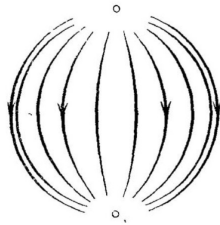
### Quality factor

$$Q \stackrel{\text{def}}{=} \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$$

$$1 < Q < \infty$$



## Resonances in spheres

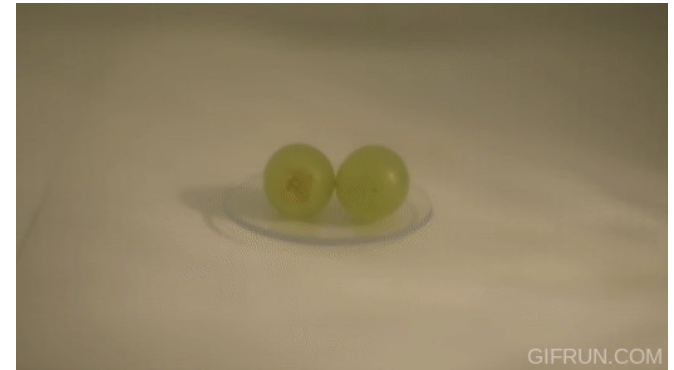


$$\frac{\lambda}{\sqrt{\epsilon}} \simeq \text{size}$$

Gustav Mie., *Annalen der physik*, 330, 377-445 (1908)

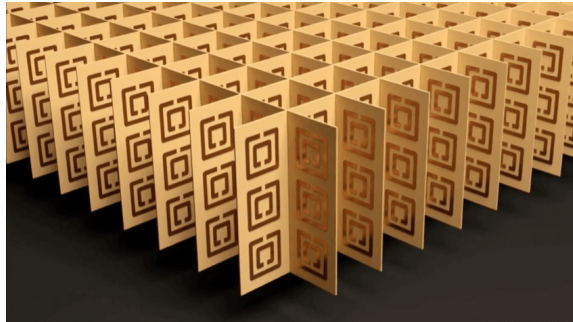
## Grapes in microwave

$$\frac{\lambda}{\sqrt{\epsilon}} \sim 1.3 \text{ cm}$$



Khattak, H.K., et al. *Soft matter*, 15(29), 5804 (2019)

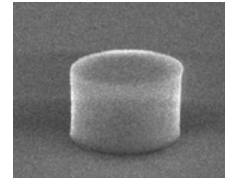
## Metamaterials



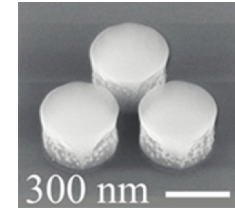
D. Smith et al. *Science* **305**, 788 (2004)

## Metastructures

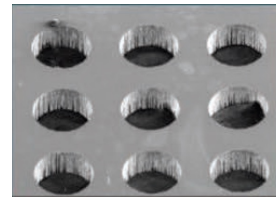
nanoresonators



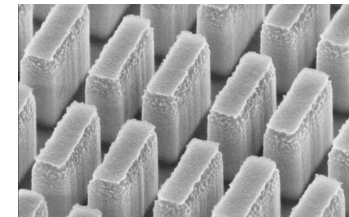
oligomers



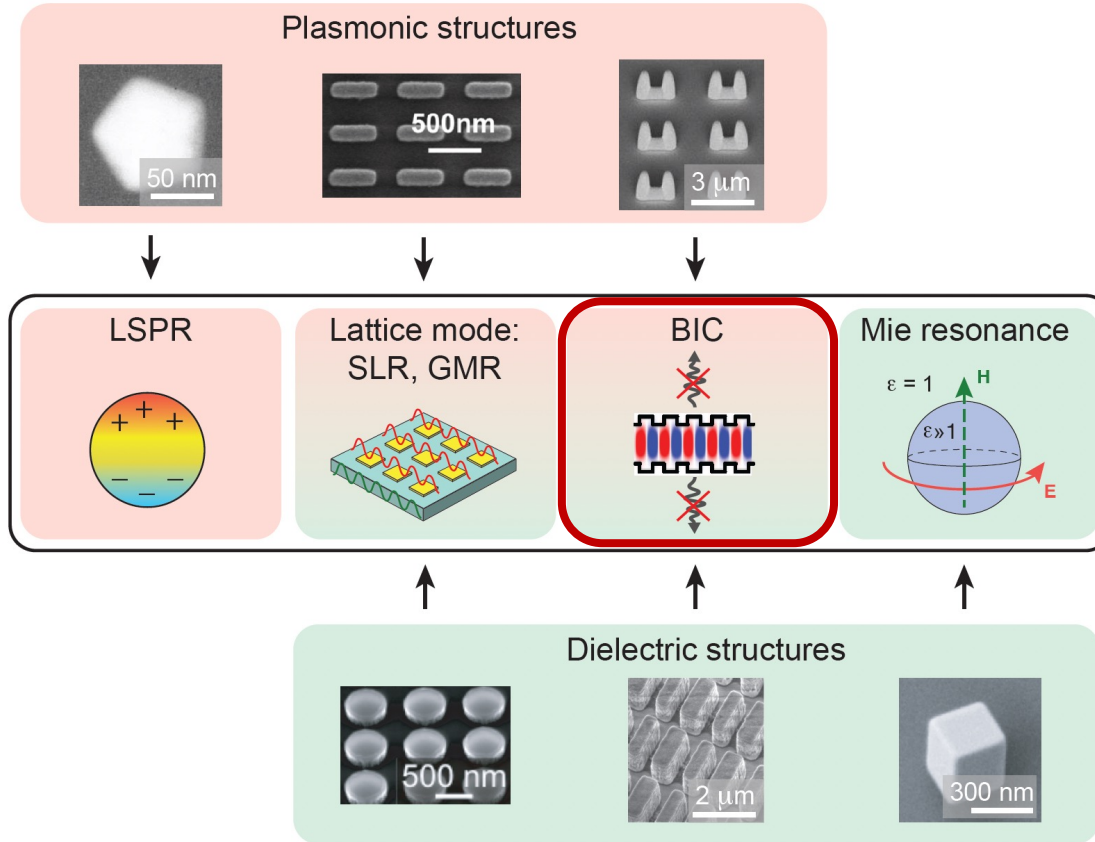
membranes



metasurfaces



K. Koshelev et al. *ACS Photonics* **8**, 102–112 (2021)





## Quasi-BIC metasurfaces

Designing a high-Q metasurface



Experimental observation



Harmonic generation



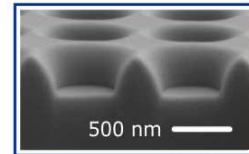
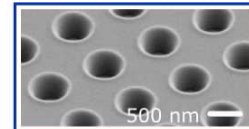
High-harmonic  
generation

## Mie voids

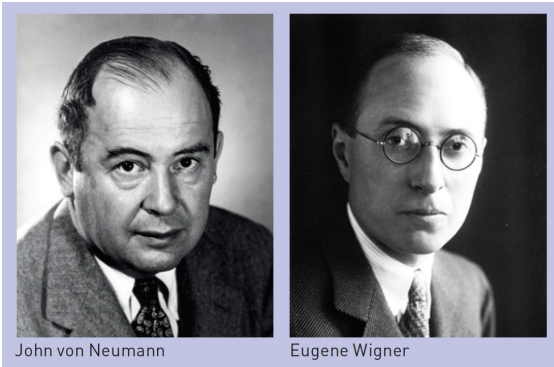
High-Q modes in visible and UV



Generation of structured colors



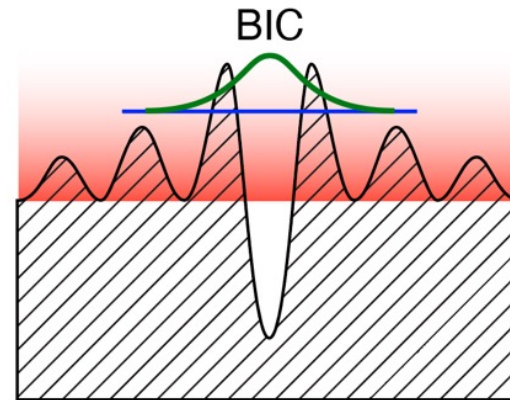
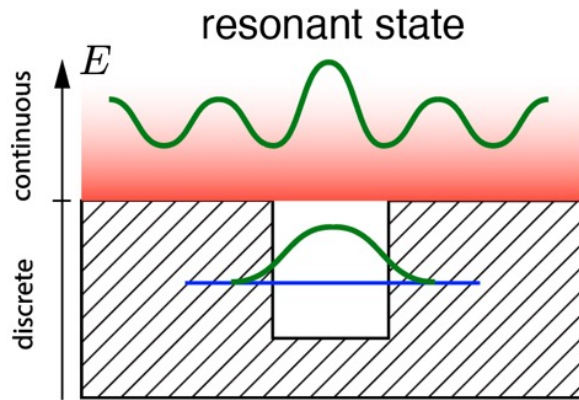
# Bound state in the continuum (BIC): history

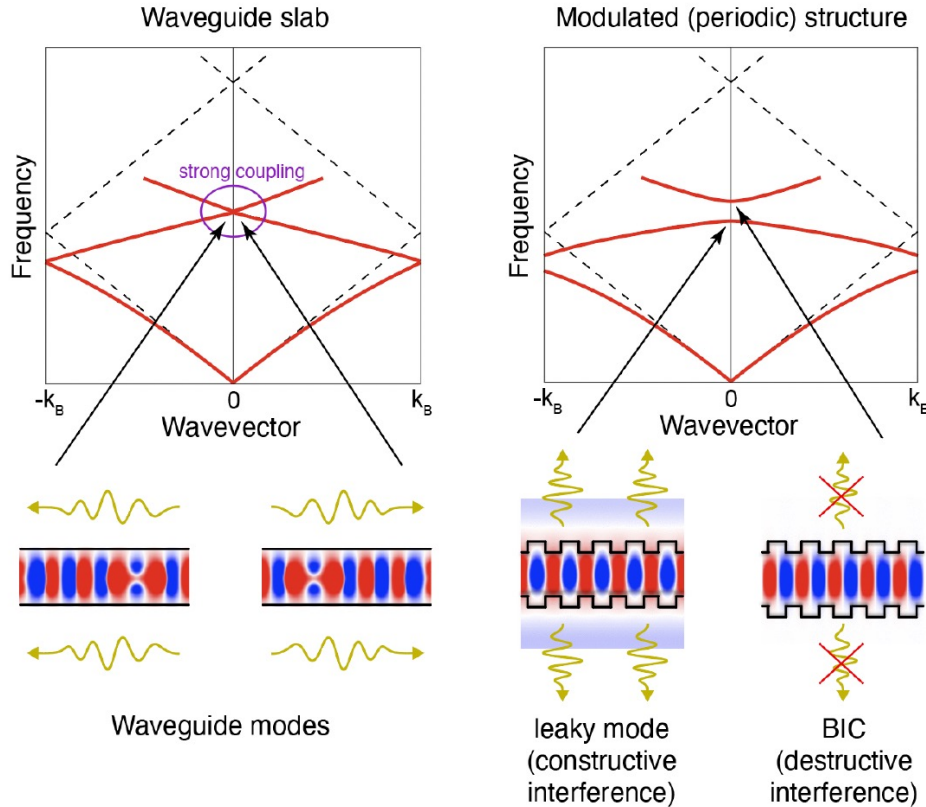


## Über merkwürdige diskrete Eigenwerte

J. von Neumann and E. P. Wigner

Physikalische Zeitschrift 30, 465–467 (1929)





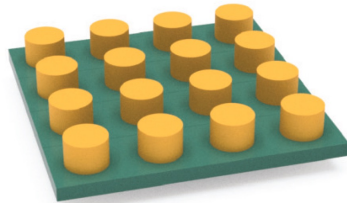
Hamiltonian of mode interaction

$$\hat{\mathcal{H}} = \begin{bmatrix} \omega_1 & \kappa \\ \kappa & \omega_2 \end{bmatrix} - i \begin{bmatrix} \gamma_1 & \pm\sqrt{\gamma_1\gamma_2} \\ \pm\sqrt{\gamma_1\gamma_2} & \gamma_2 \end{bmatrix}$$

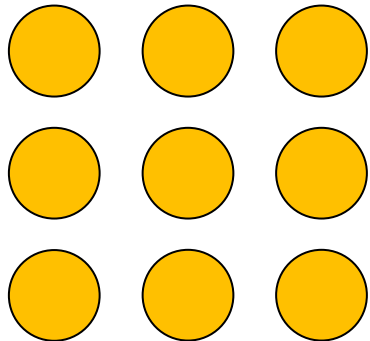
BIC condition

$$\kappa(\gamma_1 - \gamma_2) = \pm\sqrt{\gamma_1\gamma_2}(\omega_1 - \omega_2)$$

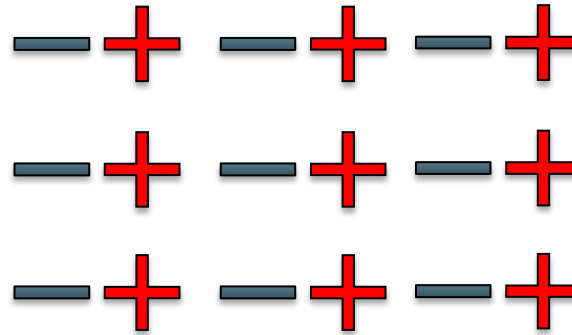
BICs can exist in symmetric structures



Symmetric unit cell  
(meta-atom)



Polarization current density  
(Electric field)



No radiation  $\rightarrow$  BICs

$$Q = \infty$$

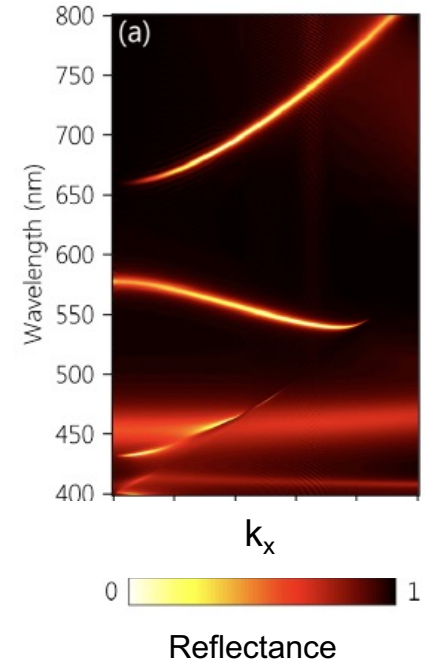


Fig. from S. Azzam et al. *PRL* 121 (2018).

Energy inside  
resonator

$$W(t) = W_0 \exp(-\gamma t)$$

Lifetime

$$t = \frac{1}{\gamma}$$

Radiation

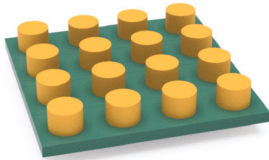
Material  
absorption

Surface  
roughness

Radiation  
(sample edges)

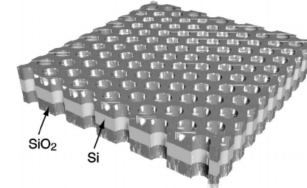
Disorder  
effects

$$\gamma = \gamma_{\text{rad}} + \gamma_{\text{abs}} + \gamma_{\text{surf}} + \gamma_{\text{edg}} + \gamma_{\text{dis}}$$



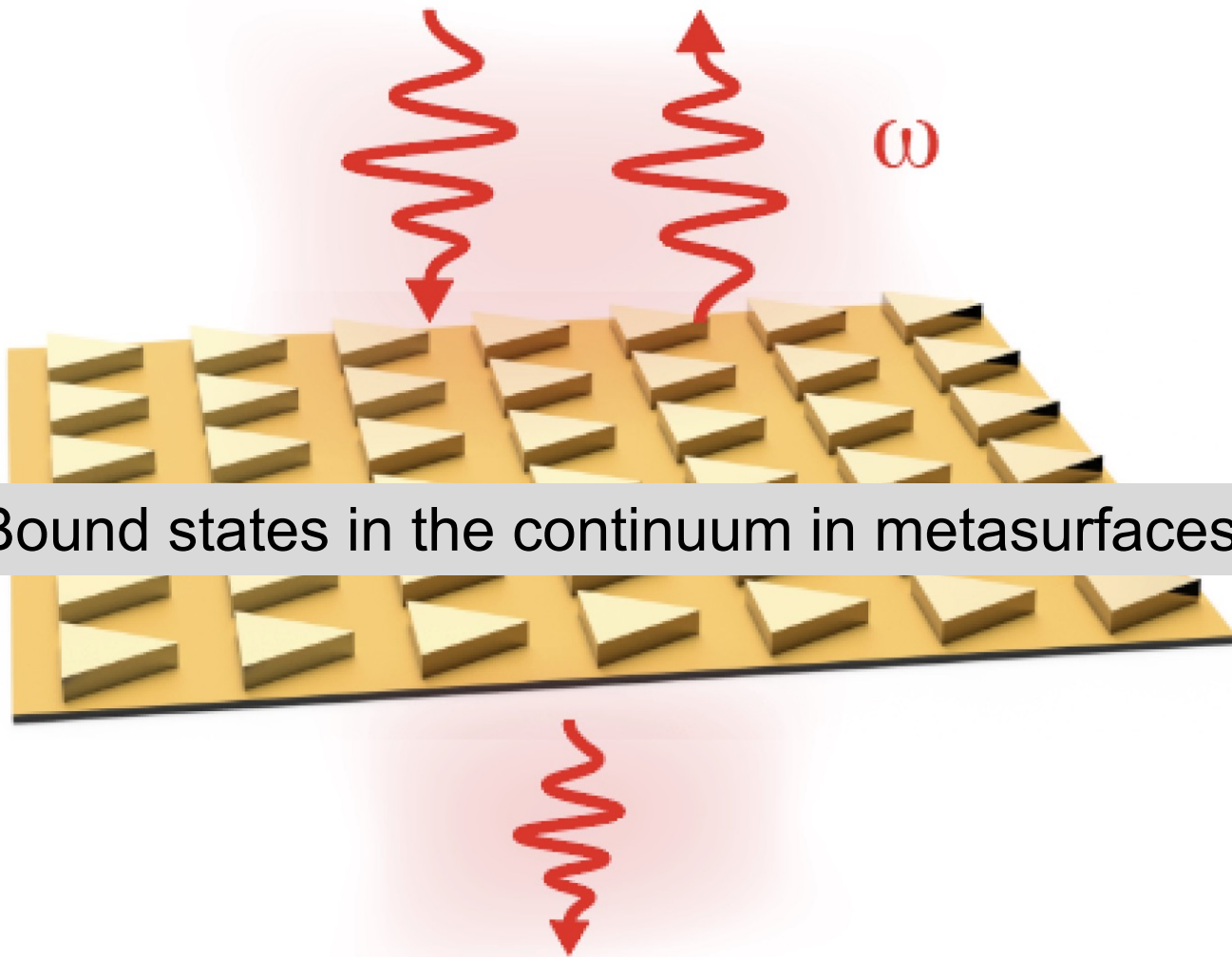
**BIC Q factor =  $10^2 - 10^4$**

Z. Liu *et al.*, *PRL* **123**, 253901 (2019)



**BIC Q factor =  $10^2 - 10^5$**

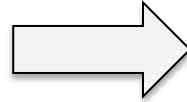
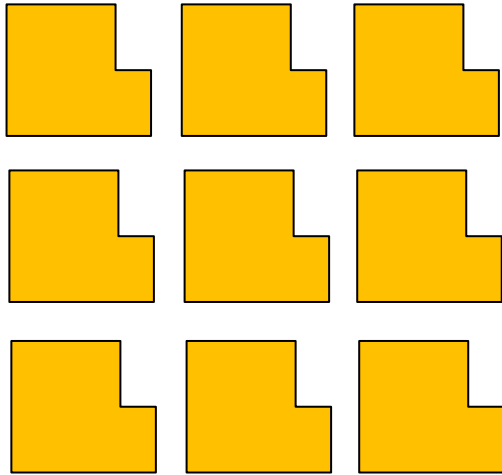
J. Jin *et al.*, *Nature* **574**, 501-504 (2019)



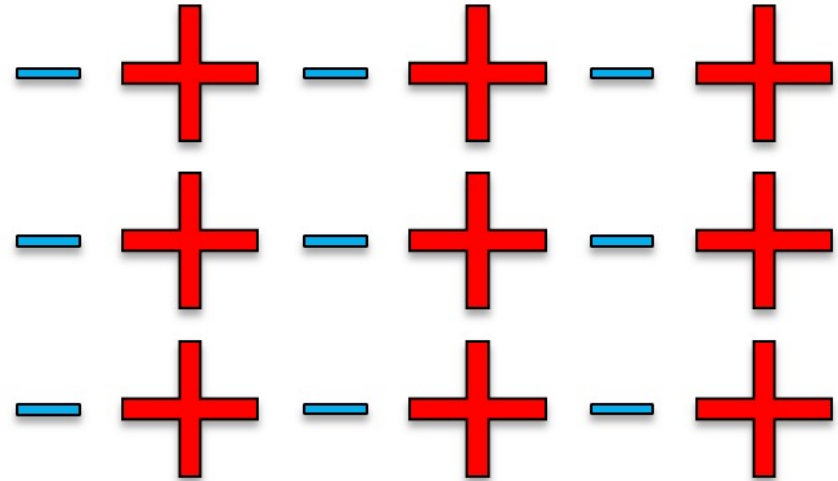
Bound states in the continuum in metasurfaces

# BICs in asymmetric metasurfaces

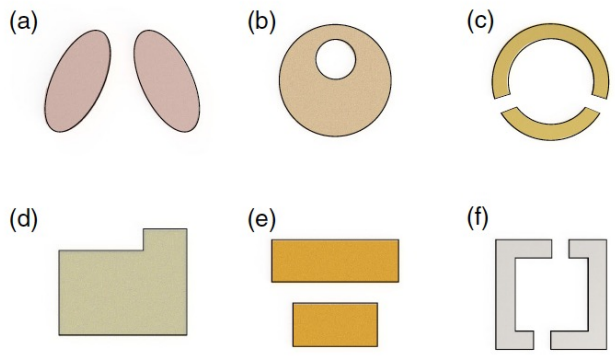
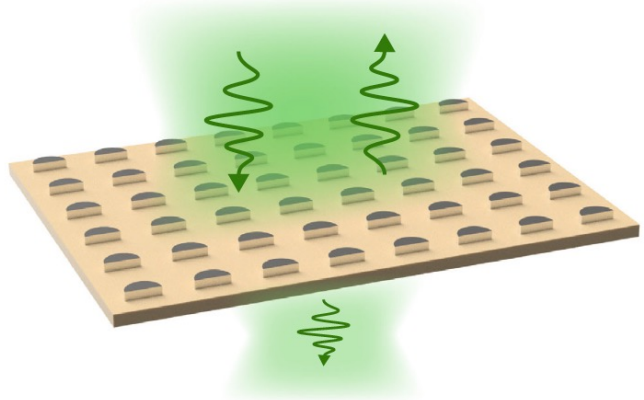
Asymmetric unit cell  
(meta-atom)



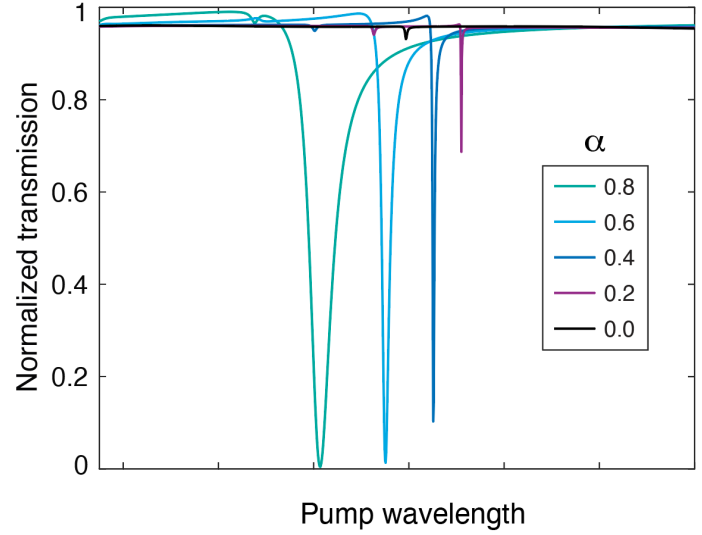
Polarization current density  
(Electric field)



Imbalance -> radiation



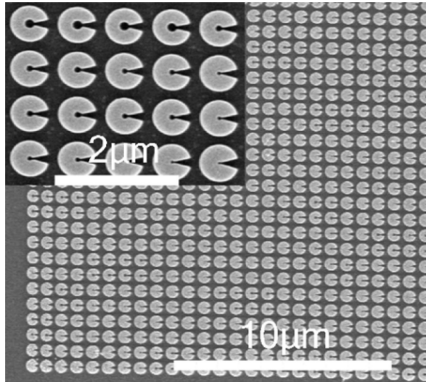
Variation of asymmetry parameter  $\alpha$



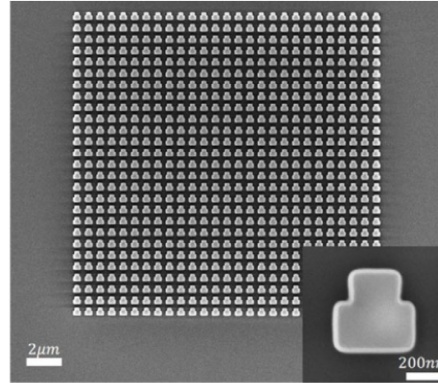
$$Q_{\text{rad}}(\alpha) \propto \alpha^{-2}$$



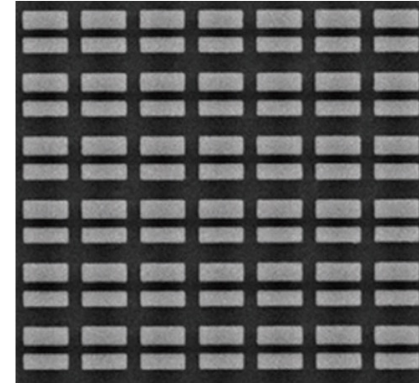
# Examples of asymmetric metasurfaces



Y. Gao *et al.* *Nano letters* **18**,  
8054-8061 (2018)



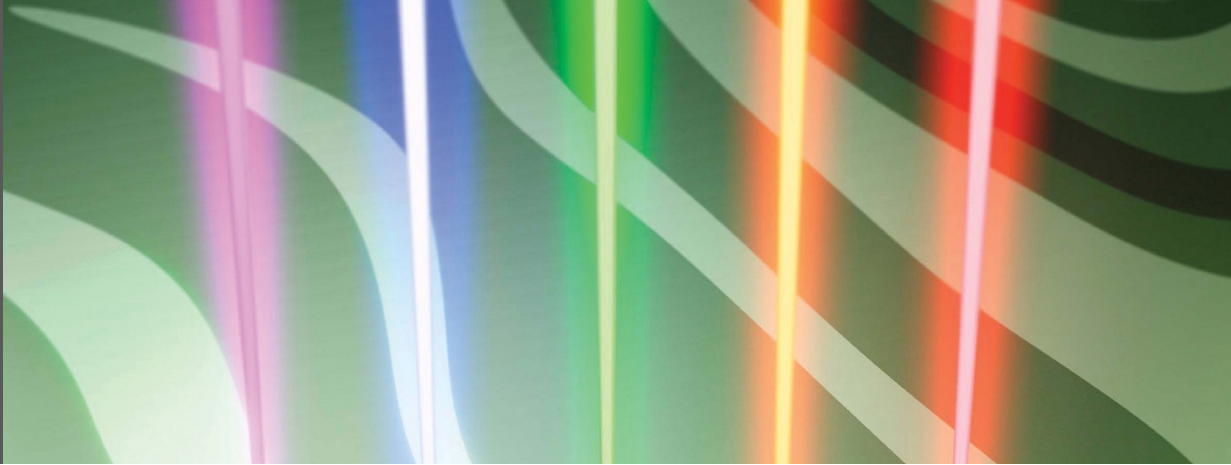
Z. Liu *et al.* *Physical Review  
Letters* **123**, 253901 (2019)



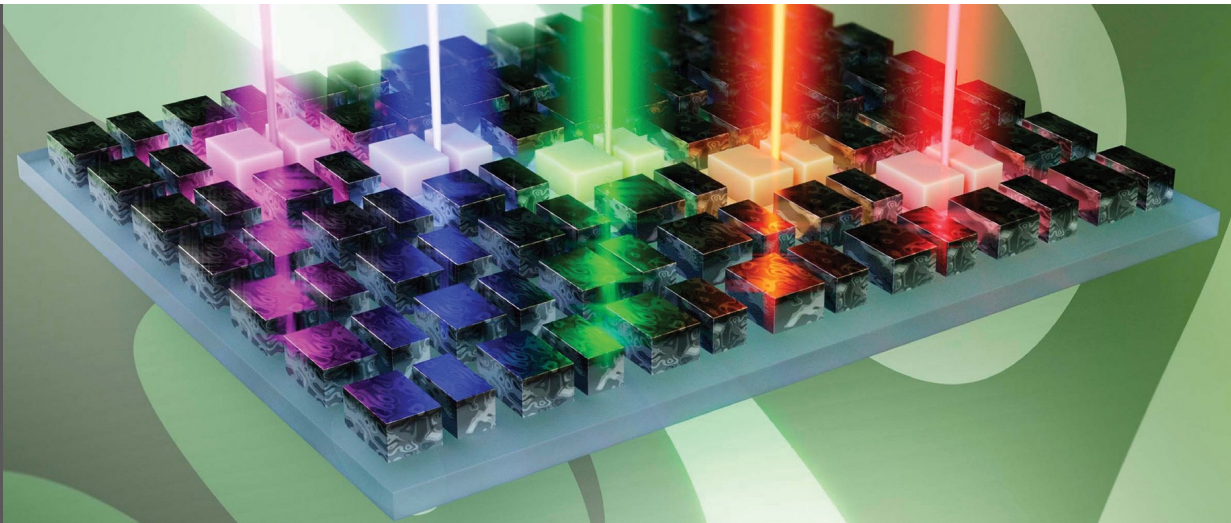
A. Ndao *et al.*  
*Nanophotonics* (2020)  
[10.1515/nanoph-2020-0008](https://doi.org/10.1515/nanoph-2020-0008).

**BIC Q factor = 18500**

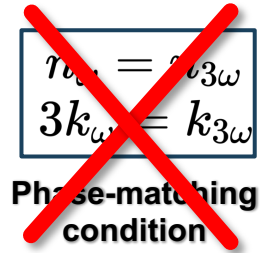
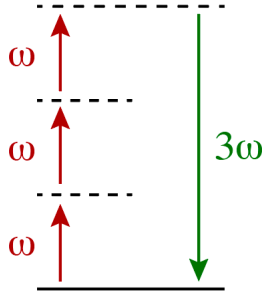
**BIC Q factor = 750**



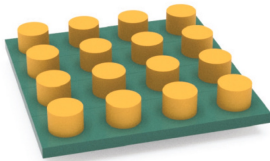
## Nonlinear metasurfaces with BICs



## Nonlinear frequency generation in macroscopic media



## Nonlinear generation in resonant nanostructures

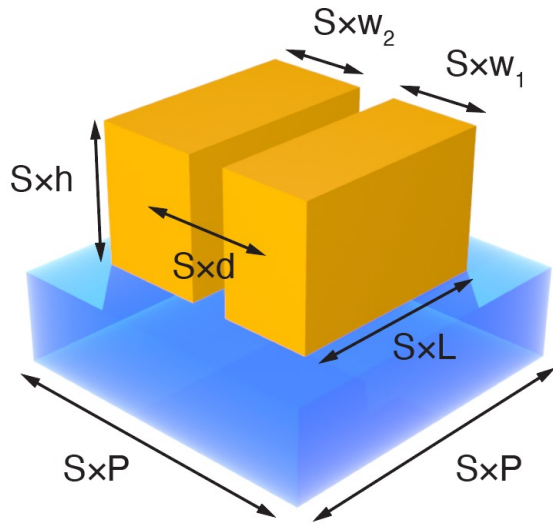


### Resonances in metasurfaces

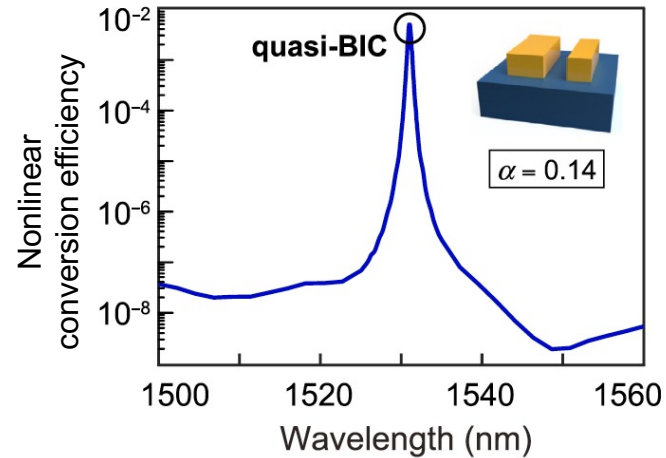
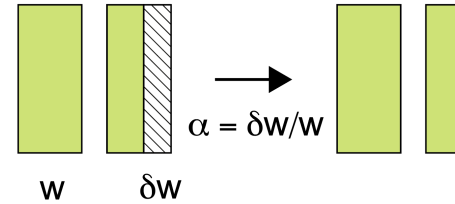
$$\frac{\text{generated nonlinear power}}{\text{pump power}} \propto Q^3 \quad 10^{-6} - 10^{-3} \%$$

# Nonlinear photonics with BICs: universal design

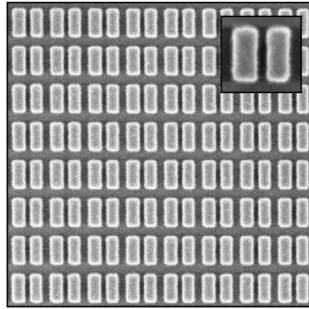
One universal design



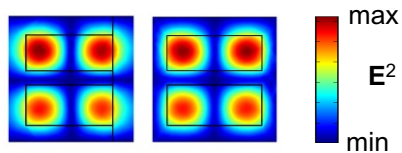
Breaking meta-atom symmetry



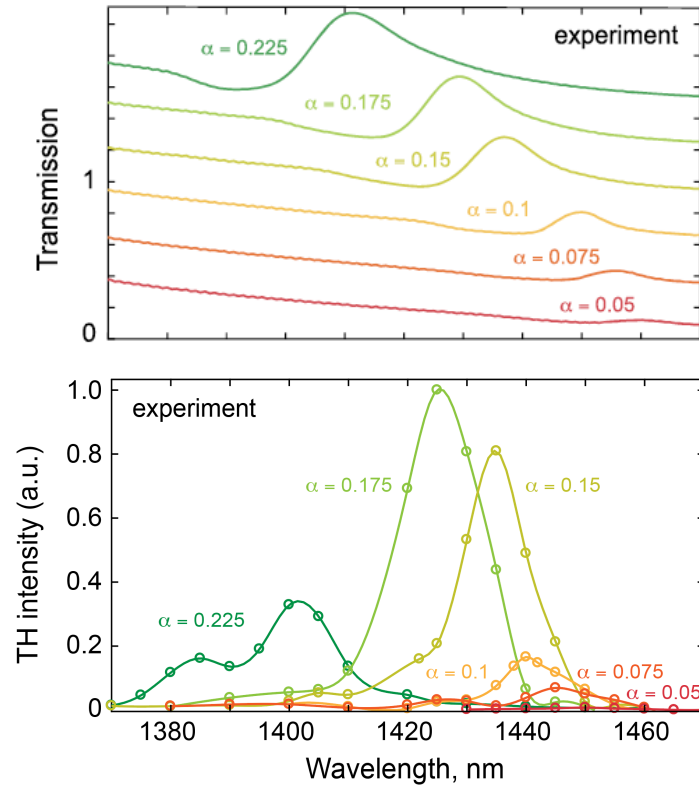
Si metasurface

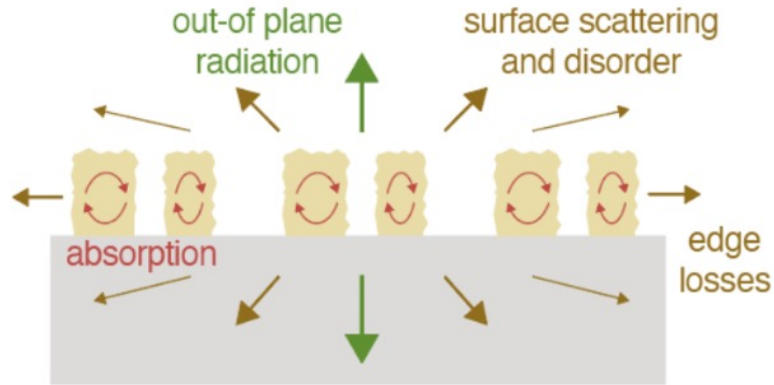


BIC resonance



$Q \sim 100$





Field enhancement

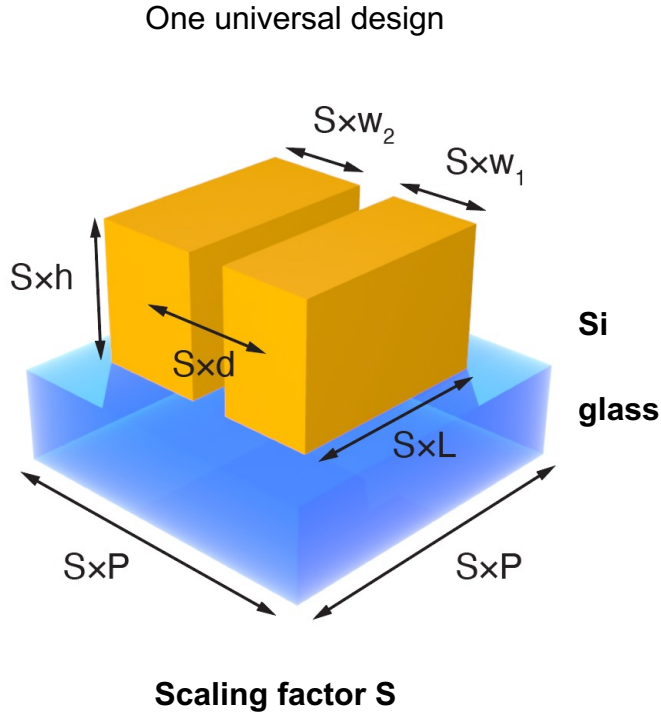
$$|\mathbf{E}|^2, |\mathbf{H}|^2 \propto \frac{Q_{\text{rad}} Q_{\text{par}}}{(Q_{\text{rad}} + Q_{\text{par}})^2}$$

$$Q_{\text{tot}}^{-1} = \underbrace{Q_{\text{rad}}^{-1} + Q_{\text{par}}^{-1}}_{\text{radiative}} + \cancel{\underbrace{Q_{\text{abs}}^{-1}}_{\text{non-radiative}}}$$

$$Q_{\text{par}}^{-1} = Q_{\text{surf}}^{-1} + Q_{\text{dis}}^{-1} + Q_{\text{size}}^{-1} + Q_{\text{sub}}^{-1}$$

Critical coupling

$$Q_{\text{rad}} = Q_{\text{par}}$$



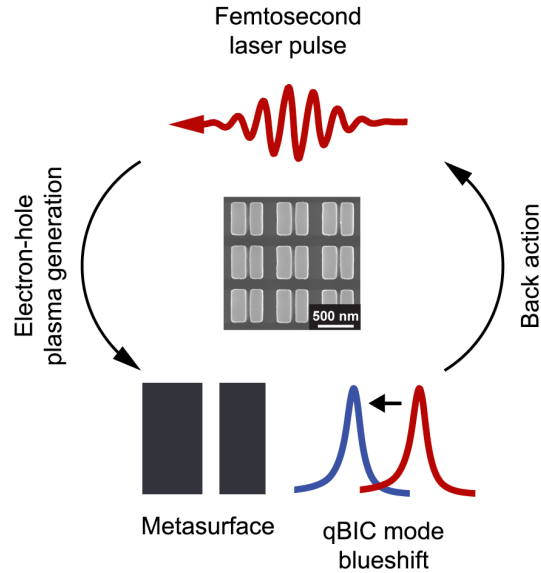
## High-Q metasurface

- Geometric scaling:  $S = 0.5 \leftrightarrow 3$
- Radiative Q factor: asymmetry

$$Q_{\text{rad}}(\alpha) \propto \alpha^{-2}$$

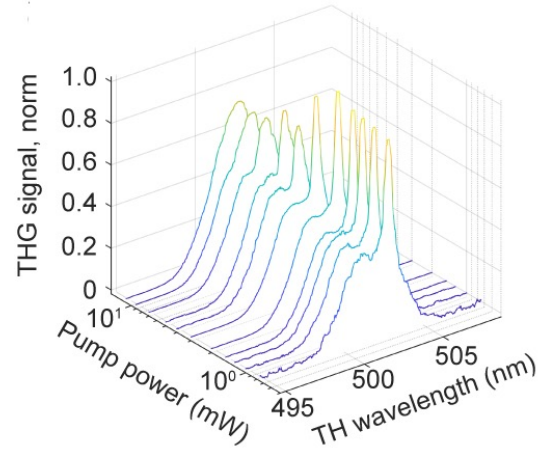
- Field enhancement: critical coupling

$$Q_{\text{rad}} = Q_{\text{par}}$$

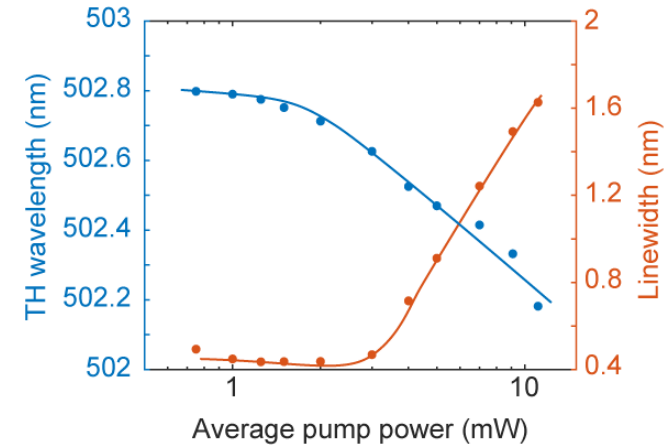


$Q \sim 900$

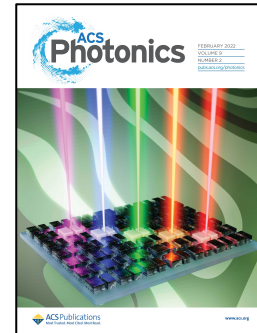
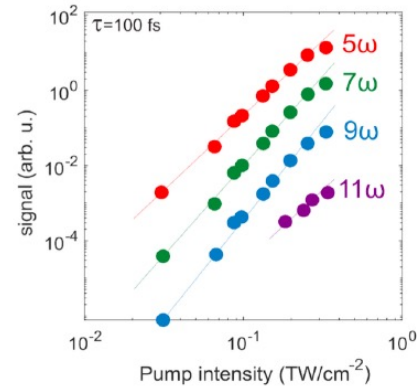
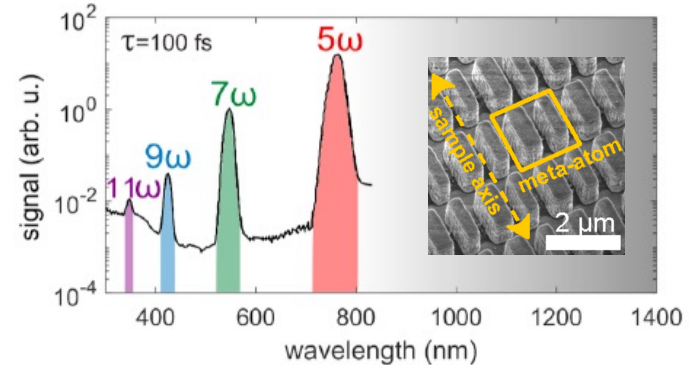
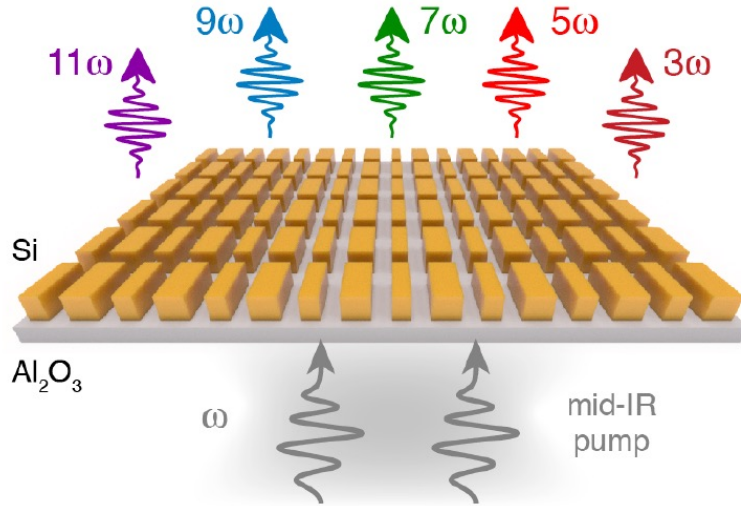
THG signal



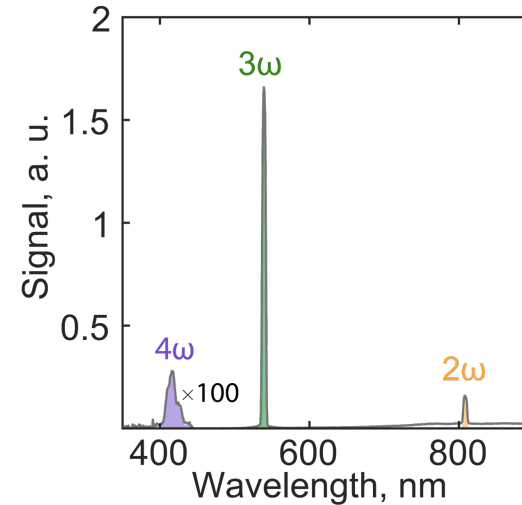
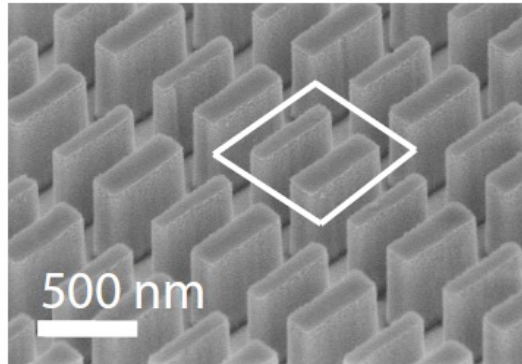
Self-action

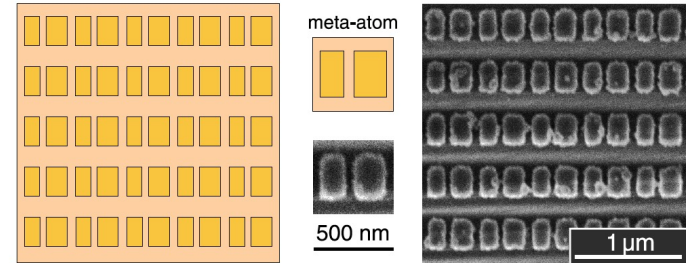
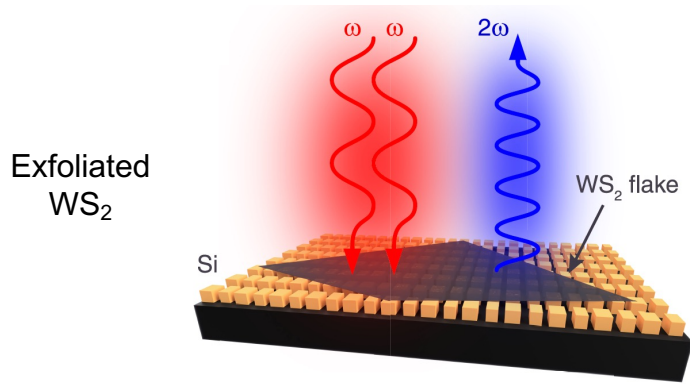




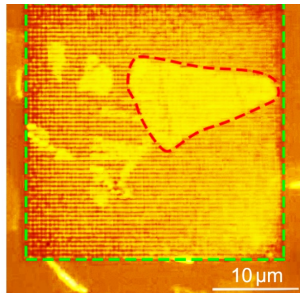


Si metasurface

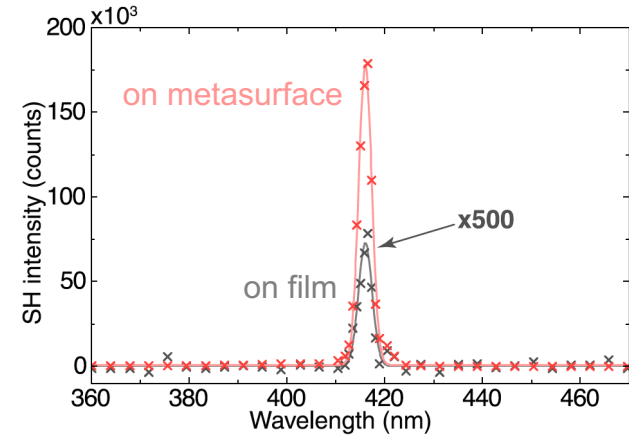
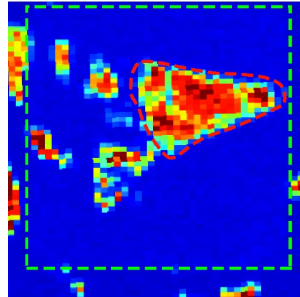




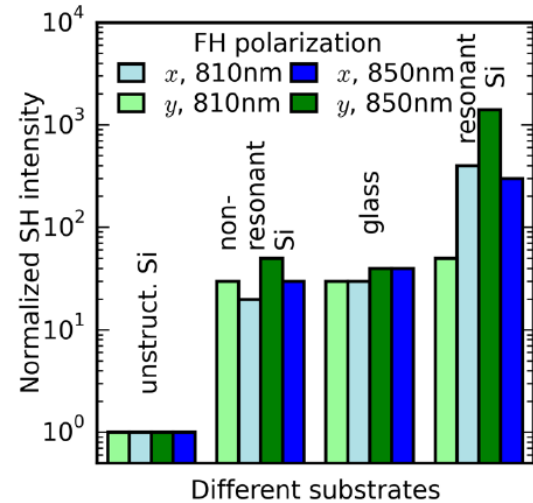
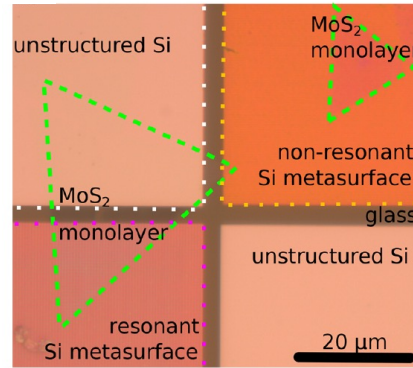
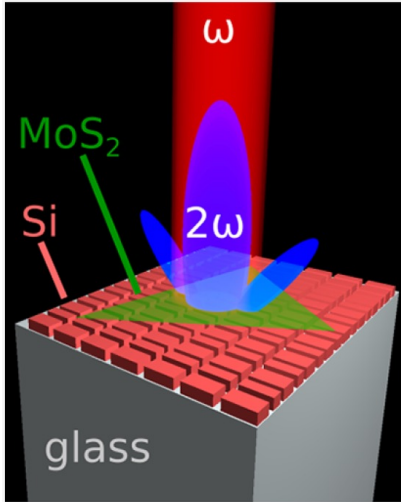
Optical image

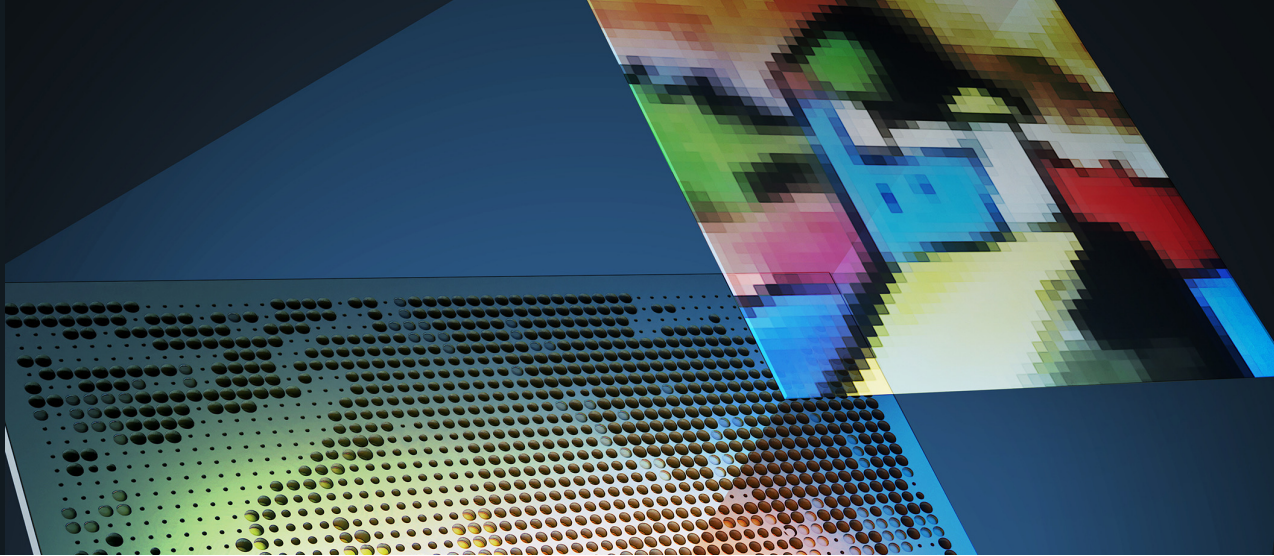


PL map

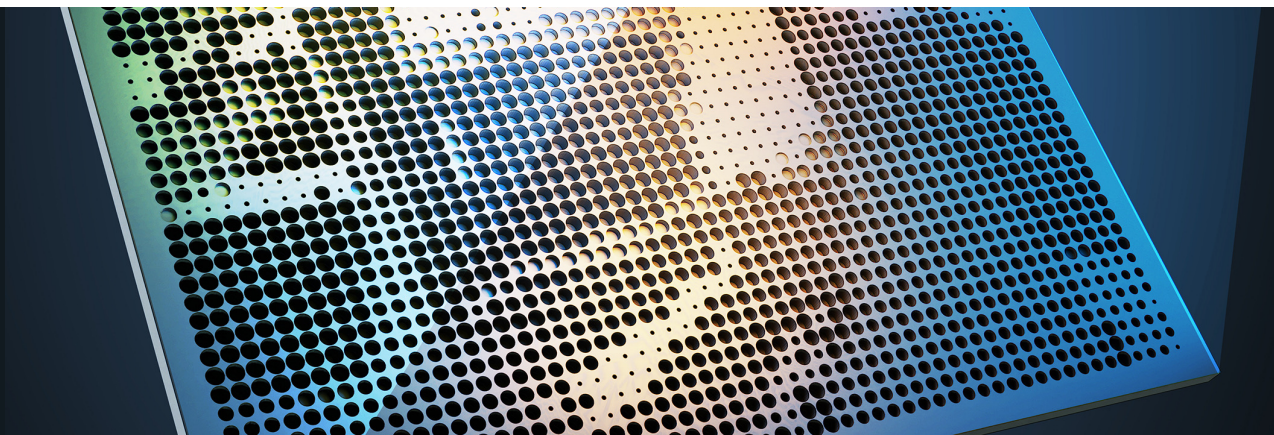


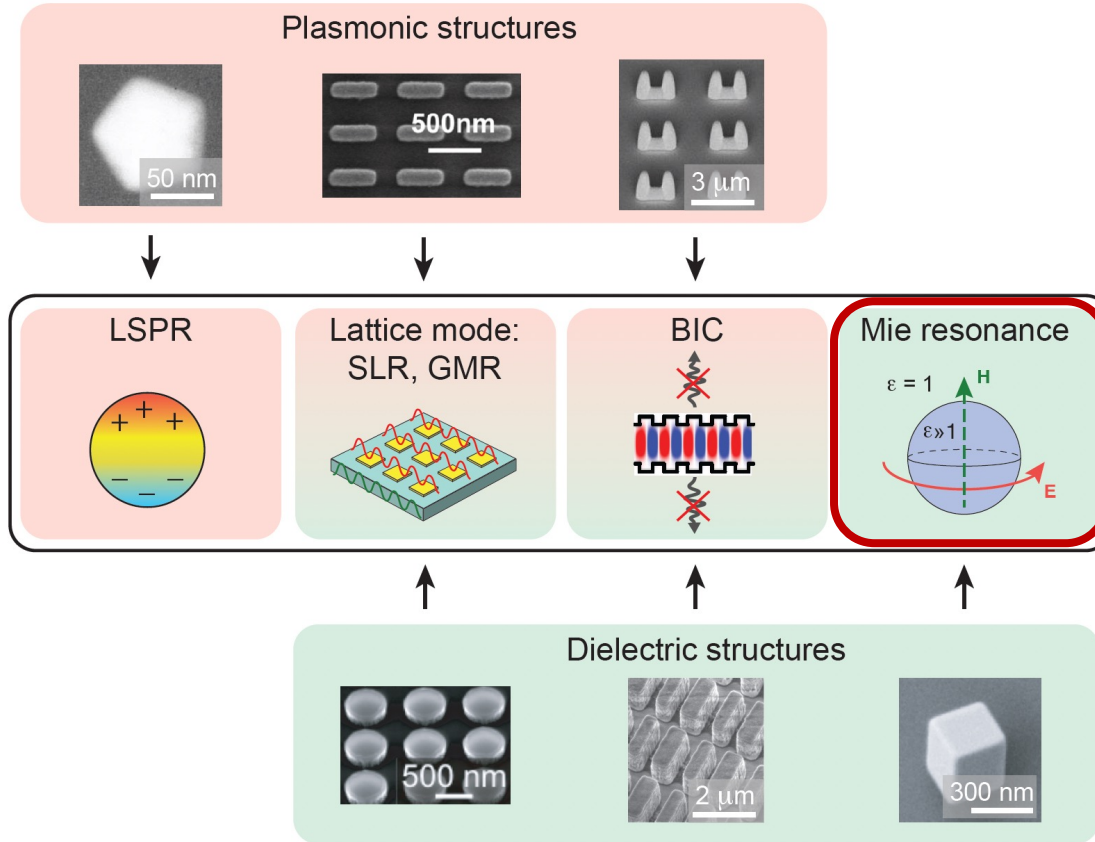
CVD-grown MoS<sub>2</sub>



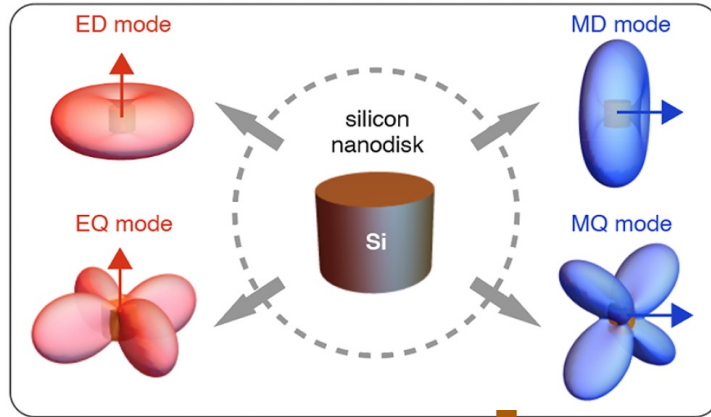


Mie void resonances: confining light in air

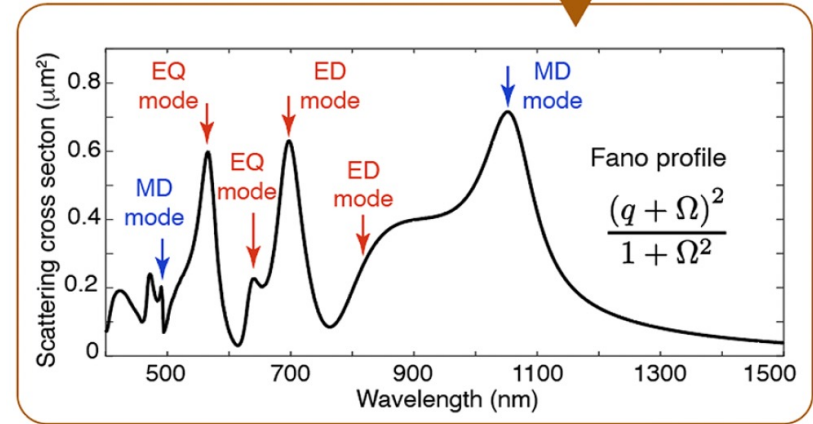




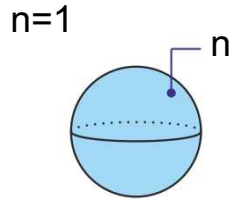
## Modes: Mie resonances



## Scattering: Fano resonances

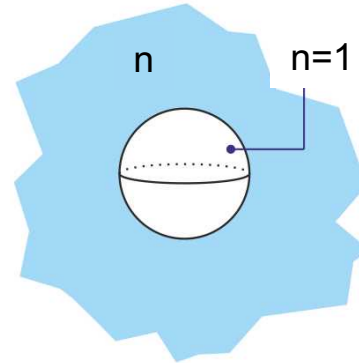


Mie sphere



G. Mie, Ann. Phys. **4**, 377 (1908)

Mie void



Maxwell's equations (TM)

$$\frac{\psi'_l(nk_0R)}{\psi_l(nk_0R)} = n \frac{\xi'_l(k_0R)}{\xi_l(k_0R)}$$

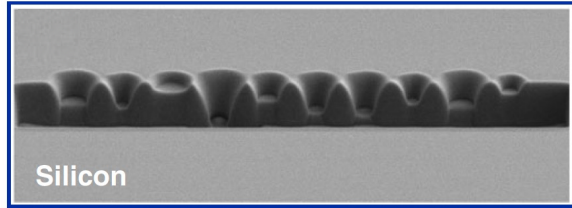
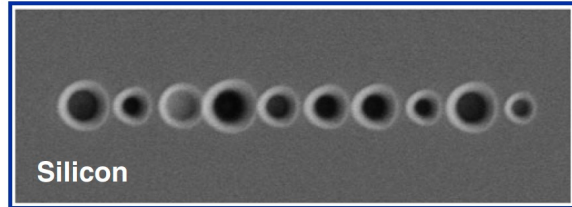
Can we solve the same Maxwell's equations?

$$\frac{\psi'_l(k_0R)}{\psi_l(k_0R)} = \frac{1}{n} \frac{\xi'_l(nk_0R)}{\xi_l(nk_0R)}$$

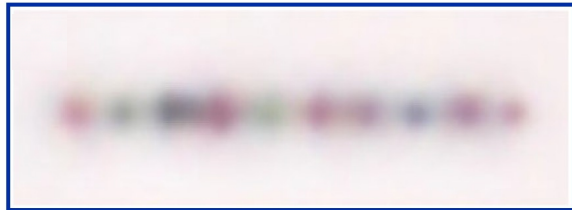




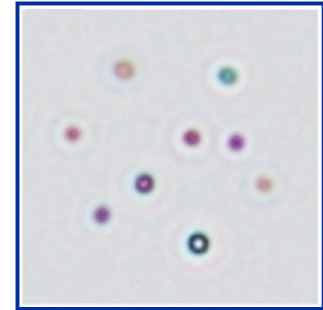
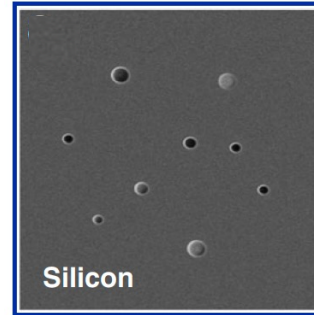
First observation of resonances in surface carvings  
(Dr Mario Hentschel, Prof Harald Giessen)



2000 nm

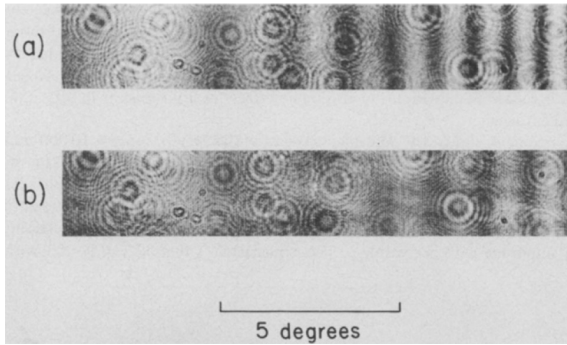


Individual nanoresonators!



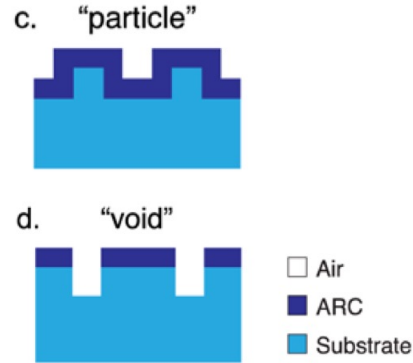
5000 nm

## Air bubbles in water



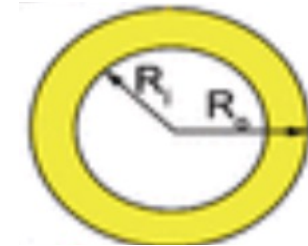
Marston, P. L. et al,  
Appl. Sci. Res. 38, 373 (1982).

## Cylindrical voids with anti-reflection coatings



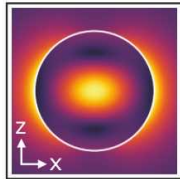
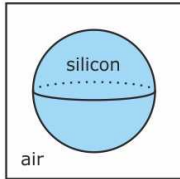
Mann, S.A. et al.  
Optics Express 19, 25729 (2011)

## Plasmonic nanoshells

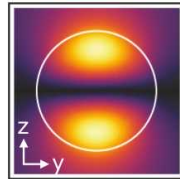


Prodan, E., Nordlander, P.,  
Nano Letters 3, 543 (2003)

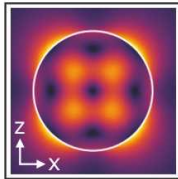
## Mie sphere



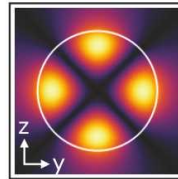
ED  
electric dipole



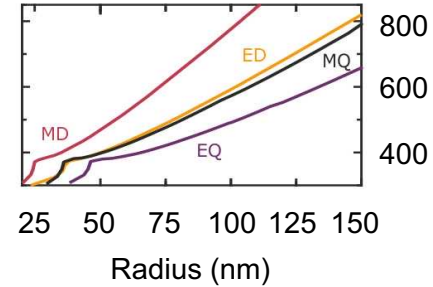
MD  
magnetic dipole



EQ  
electric quadrupole

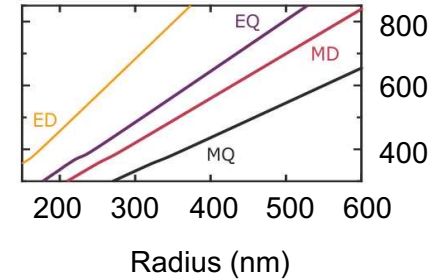
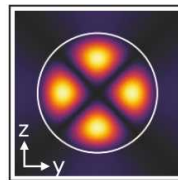
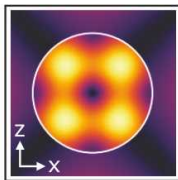
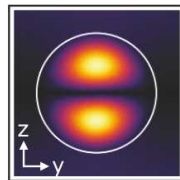
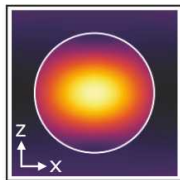
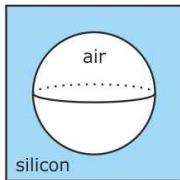


MQ  
magnetic quadrupole



Resonant wavelength (nm)

## Mie void

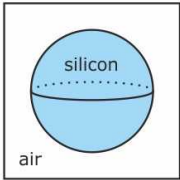


Resonant wavelength (nm)

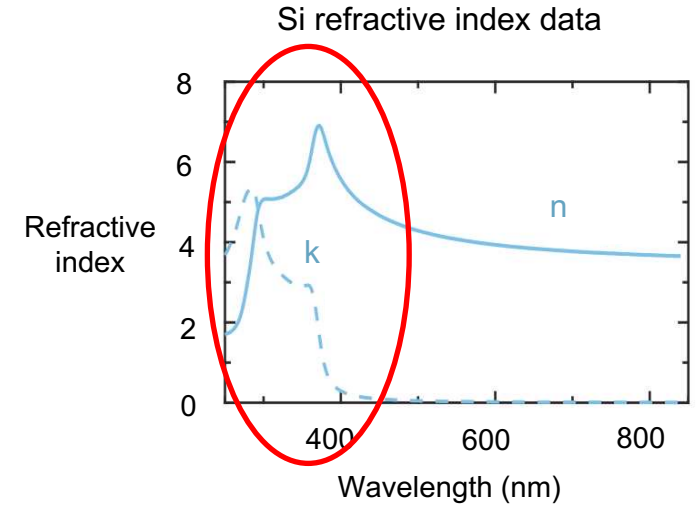
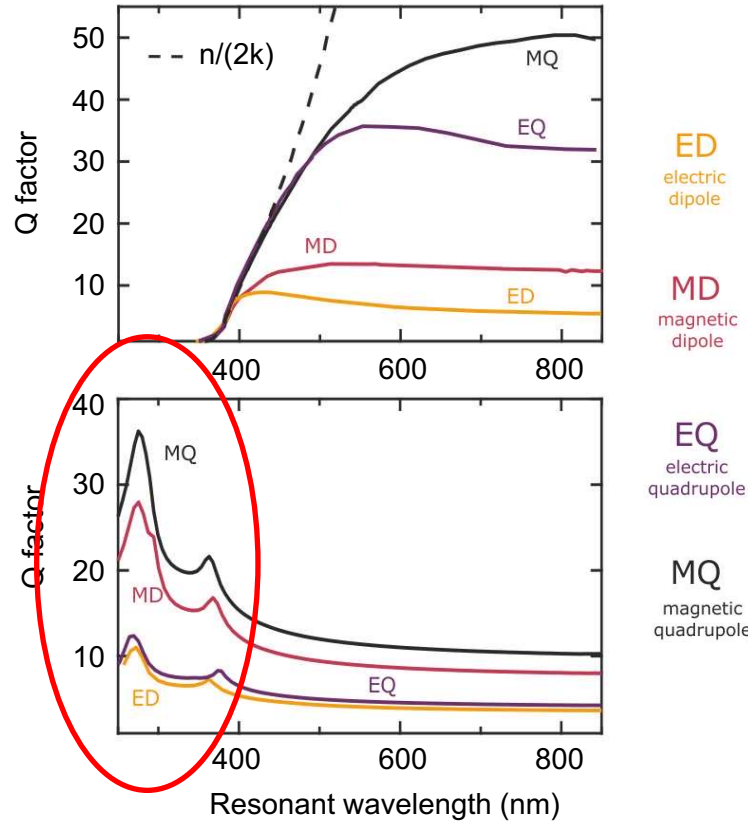
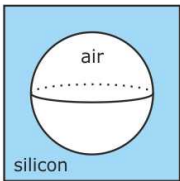
min  max  
|E| (a.u.)

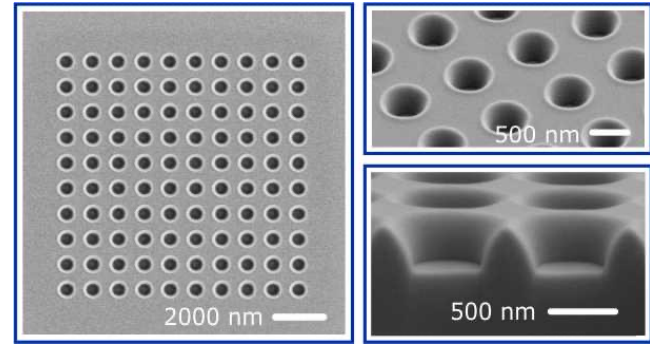
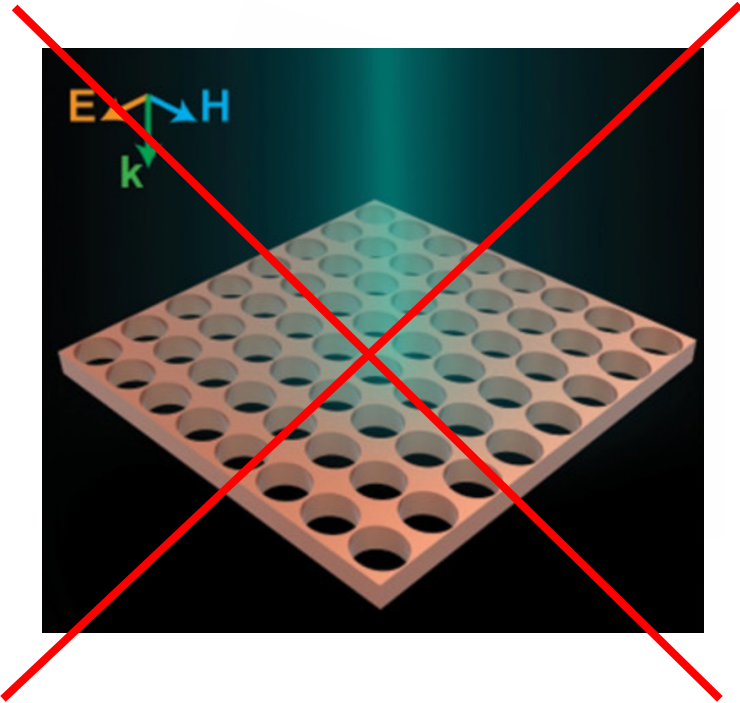


Mie sphere



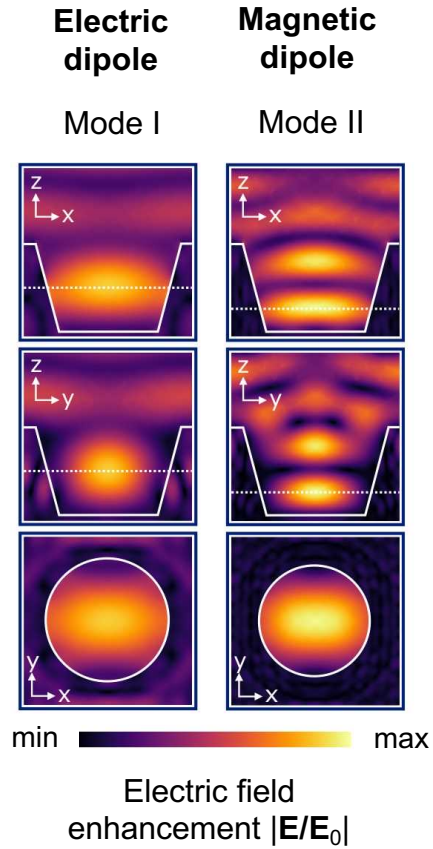
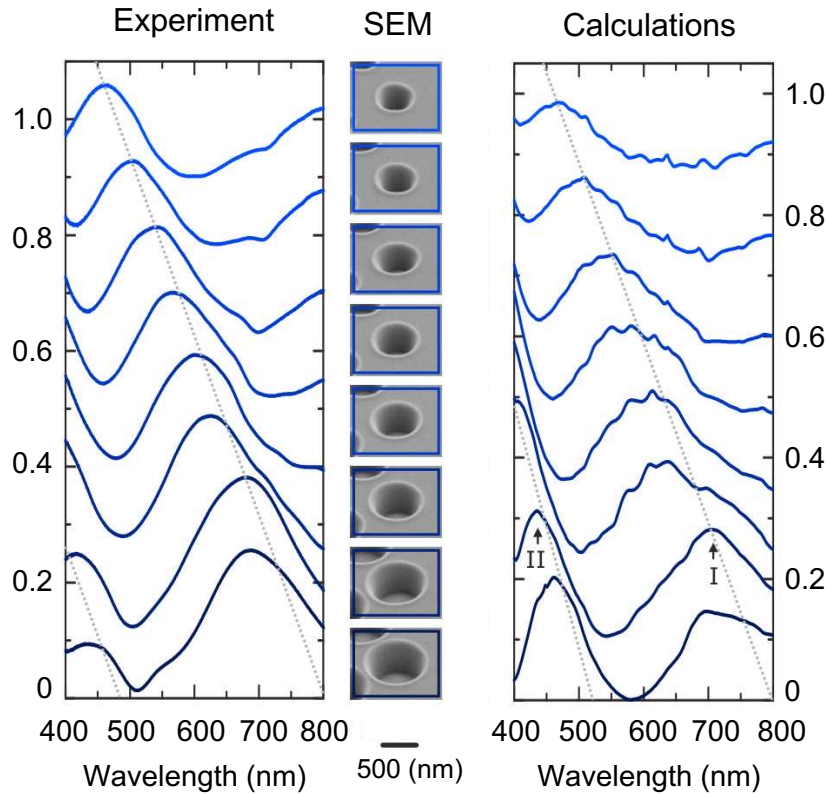
Mie void



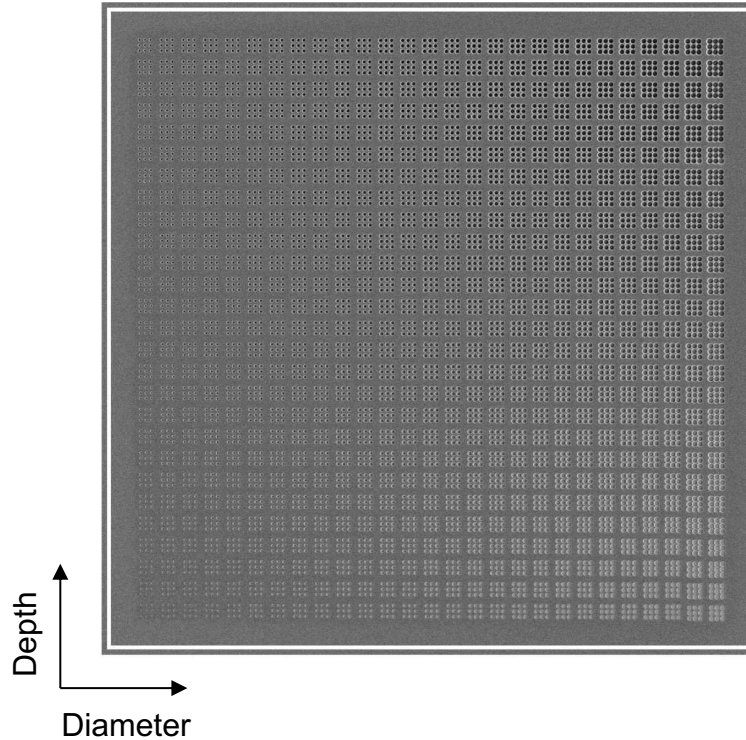


Minimal thickness is several wavelengths to prevent coupling to guided modes

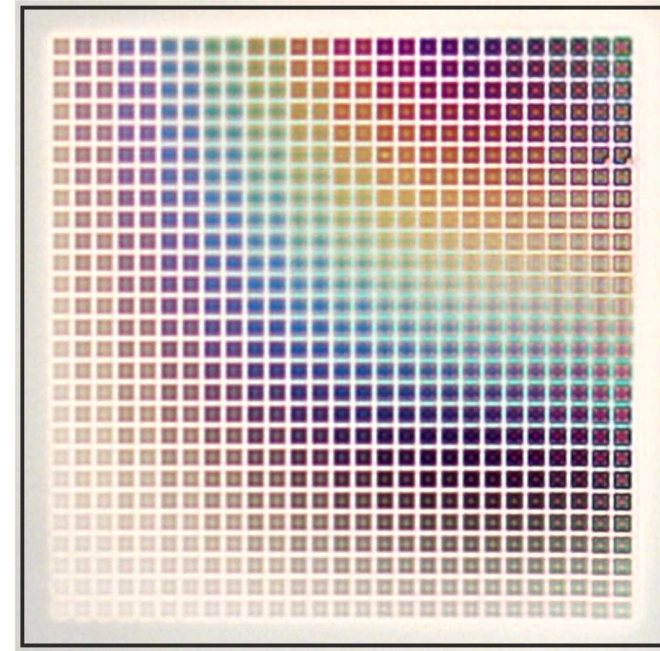
# Mie voids in scattering



SEM



Optical microscope (white light)



25  $\mu\text{m}$



## Near-IR

## Visible

## UV

Experiment

Calculation

Experiment

Calculation

Experiment

Calculation

$\lambda = 750 \text{ nm}$

$\lambda = 450 \text{ nm}$

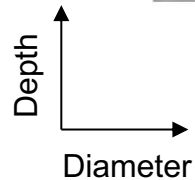
$\lambda = 265 \text{ nm}$

$\lambda = 800 \text{ nm}$

$\lambda = 500 \text{ nm}$

$\lambda = 313 \text{ nm}$

Depth  
Diameter



Number of maxima = number of modes







Wassily Kandinsky  
Painting: Improvisation 9 (1910)



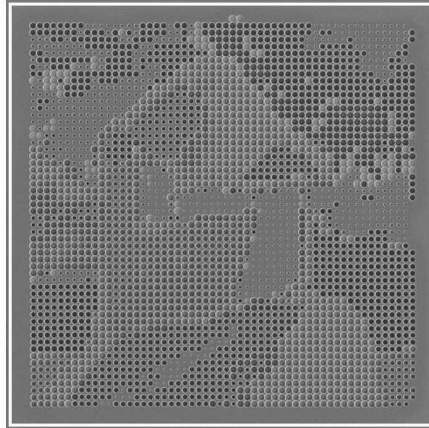
Video by M. Hentschel

100  $\mu\text{m}$



# Mie voids for color generation

Original  
image

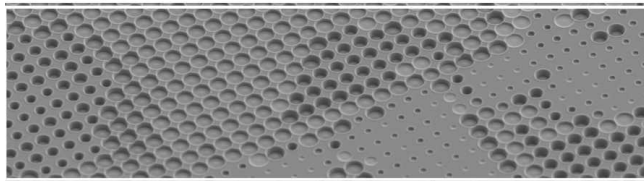


Experiment



15 µm

15 µm



5 µm

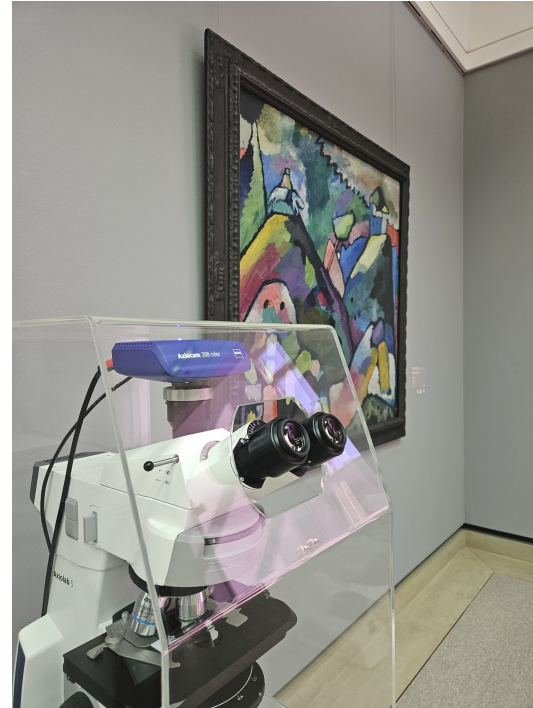


Resolution - 36000 dpi

**Editorial:** Oliver Graydon. Mie voids generate miniature artworks. *Nature Photonics* 17, 133 (2023)

# Nano-Kandinsky in Stuttgart Gallery

Staatsgalerie Stuttgart, Germany



Exposition was open September – December 2023

## Conclusions

- BIC in meta-structures – versatile platform for engineering resonances
- Design of BIC in metasurfaces: asymmetry & critical coupling
- Harmonic generation enhancement: >1000 times stronger
- Mie voids: new platform for dielectric nanophotonics

## Relevant papers

- K. Koshelev et al., Optics and Photonics News 31, 38 (2020)
- K. Koshelev et al., ACS Photonics 8, 102–112 (2021)
- K. Koshelev et al., Physics-Uspekhi 66, 494 (2023)
- M. Hentschel, K. Koshelev et al. Light: Sci. Appl. 12, 3 (2023)

